

# ORION S-Band Data Acquisition for S-X Calibration

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*To calibrate for ionospheric charged particle effects, ORION will include both X- and S-band data links. This report studies the performance of S-band receiver and the optimum allocation of data volume between S- and X-band observations. It is found that the requirements on the S-band receiver are not very stringent.*

## I. Introduction

Covariance analyses have been performed to estimate the effects of possible error sources on ORION baseline determination (Ref. 1). In the analyses, the ORION observations were assumed to be taken only at X-band frequencies. In other words, no S-X calibration was considered; the effects of ionospheric charged particles were assumed partially calibrated for by a global model. Such a global model requires an independent measurement of the ionosphere, such as by Faraday rotation measurement, and relies on a mapping scheme to obtain line-of-sight calibrations.

With the inclusion of an S-band link, ionospheric calibration becomes simple and complete through the combination of S- and X-band observations. However, it is costly to have an S-band receiver performing as well as the primary X-band receiver. An S-band receiver with a system temperature a factor of 2 higher than that of the X-band receiver is available at a much lower cost. In this report, we shall compare the performance of S-band receivers with different system temperatures. Also, the optimum allocation of data volume between S- and X-band observations will be studied.

## II. Analysis

In the following analysis, we shall assume that all 14 pairs of the Mark III VLBI data channels are available for maintaining high delay precision. In other words, no channel pairs are to be placed at reduced span bandwidths for ambiguity resolving. It appears that the smaller span bandwidth of 100 MHz at S-band (in comparison with 400 MHz at X-band) may be used to resolve the ambiguity of the X-band delays. Parameters to be used in the analysis are listed as follows:

Quasar correlated flux density  $S = 2$  janskys

Span bandwidth  $f_{span} = 400 \times 10^6$  Hz (X-band)  
 $= 100 \times 10^6$  Hz (S-band)

Total channel bandwidth  $B = 14 \times (2 \times 10^6)$  Hz

Integration time per observation  $T = 600$  s

Antenna efficiencies:

At co-site  $e_1 = 0.5$  (S and X)

At mobile site  $e_2 = 0.5$  (X-band)  
 $0.4$  (S-band)

Antenna diameters:

At co-site  $D_1 = 9$  m or 26 m

At mobile site  $D_2 = 5$  m

System temperatures:

At co-site  $T_1 = 40$  K (S and X)

At mobile site  $T_2 = 100$  K (X-band)  
60 – 240 K (S-band)

The  $1-\sigma$  error of delay observation due to receiver system noise is (Ref. 1)

$$\sigma_\tau = \frac{\sqrt{2}c}{2\pi f_{span}} \left( \frac{1}{SNR} \right) \quad (1)$$

where SNR is the fringe signal-to-noise ratio,

$$SNR = 2.05 \times 10^{-4} S D_1 D_2 \left( \frac{e_1 e_2 B T}{T_1 T_2} \right)^{1/2} \quad (2)$$

and  $c$  is the speed of light. When all 14 pairs of the data channels are used for X-band only, the delay observations will have a  $1-\sigma$  error of 0.89 cm due to the system noise with a 9-meter antenna at the co-site. To account for different system parameters, the  $1-\sigma$  delay error can be written as

$$\sigma_\tau = 0.89 \left( \frac{400}{f_{span}} \right) \left( \frac{9}{D_1} \right) \left( \frac{T_2}{100} \right)^{1/2} \left( \frac{0.5}{e_2} \right)^{1/2} \text{ cm} \quad (3)$$

where  $f_{span}$  is in MHz and  $D_1$  in meters.

A covariance analysis following Ref. 1 shows that an error  $\sigma_\tau = 1$  cm results in a 3-D baseline error of 1.3 cm:

$$\epsilon_{BL} = 1.3 \sigma_\tau \quad (4)$$

when solving for 16 parameters (three baseline components, epoch offset and 12 segments of clock rate offsets) from 32 observations. On the other hand, the effect of an uncalibrated ionosphere on baseline determination is estimated to be about 4 cm (at X-band) per 1000-km baseline. The root-sum-square (RSS) baseline errors due to system noise and to ionospheric effects are shown in Fig. 1 as slanted curves for  $D_1 = 9$  m and 26 m.

When part of the 14 pairs of data channels are allocated to S-band observations, S-X calibration can be performed to

remove the ionospheric effects. However, system noise effects will be increased, partly due to the decrease in data volume at X-band and partly due to additional effects from S-band observations. The S-X calibrated delay observable can be expressed as a linear combination of X- and S-band observations:

$$\tau_{s-x} = \left( \frac{1}{1-\alpha^2} \right) \tau_x - \left( \frac{\alpha^2}{1-\alpha^2} \right) \tau_s \quad (5)$$

$$\div 1.08 \tau_x - 0.08 \tau_s$$

where  $\alpha = 3/11$  is the ratio of S- and X-band frequencies. The effect of system noise on such delay observable is then

$$\sigma_{\tau s-x}^2 = 1.08^2 \sigma_{\tau,x}^2 + 0.08^2 \sigma_{\tau,s}^2 \quad (6)$$

An attempt to reduce  $\sigma_{\tau,x}$  calls for a larger data volume to be allocated to X-band observations; this leaves a smaller data volume for S-band and thus increases  $\sigma_{\tau,s}$  and vice versa. Hence a compromise between S- and X-band data volume allocation needs to be made to keep  $\sigma_{\tau,s-x}$  low.

Let the fraction of data volume allocated to X-band be  $V_x$  and that to S-band be  $V_s = 1 - V_x$ ; then

$$\sigma_{\tau,x}^2 = \left( \frac{1}{V_x} \right) \sigma_{\tau,x\text{-only}}^2 \quad (7)$$

$$\sigma_{\tau,s}^2 = (20) \left( \frac{T_{2,s}}{100} \right) \left( \frac{1}{1-V_x} \right) \sigma_{\tau,x\text{-only}}^2 \quad (8)$$

where the factor 20 accounts for different  $f_{span}$  and  $e_2$  for the S-band receivers. With the substitution of Eqs. (7) and (8), Eq. (6) can be written as

$$\frac{\sigma_{\tau,s-x}^2}{\sigma_{\tau,x\text{-only}}^2} = 1.08^2 \left( \frac{1}{V_x} \right) + 0.08^2 \left( \frac{T_{2,s}}{5} \right) \left( \frac{1}{1-V_x} \right) \quad (9)$$

which is minimum when

$$V_x = \frac{1.08}{1.08 + 0.08 (T_{2,s}/5)^{1/2}} \quad (10)$$

with a minimum value of

$$\frac{\sigma_{\tau,s-x}^2}{\sigma_{\tau,x-only}^2} = \left[ 1.08 + 0.08 \left( \frac{T_{2,s}}{5} \right)^{1/2} \right]^2 \quad (11)$$

In other words, the 3-D baseline error due to the system noise in both S- and X-band receivers will be

$$\epsilon_{BL} = (1.3) (0.89) \left( \frac{9}{D_1} \right) \left[ 1.08 + 0.08 \left( \frac{T_{2,s}}{5} \right)^{1/2} \right] \quad (12)$$

Figure 2 shows the variations of baseline errors as functions of  $V_x$ . Since there are 14 pairs of data channels,  $V_x$  can have a value only of integral multiples of  $1/14$ . Hence the *theoretical* minimum 3-D baseline error  $\epsilon_{BL}$  of (12) is not always attainable. The *realizable* minimum 3-D baseline error is calculated from (9), with  $V_x$  calculated from (10) but quantized to the nearest integral multiples of  $1/14$ . However, as shown in Fig. 2, the sensitivity of the error to  $V_x$  is small near the optimum  $V_x$ . Thus, Eq. (12) is very close to the realizable minimum 3-D baseline error due to system noise.

Another concern is the requirement of exceeding a minimum signal-to-noise ratio ( $\text{SNR} > 7$ ) for phase detection. Such a criterion can easily be met with a 26-meter antenna at the

co-site. With  $D_1 = 9$  meters and  $T_{2,s} = 240$  K, an  $\text{SNR} > 7$  at S-band calls for  $V_s > 0.41$ . The closest realizable  $V_s > 0.41$  is  $6/14$ , leaving  $V_x = 8/14 = 0.57$  instead of the optimum  $0.66$  as calculated from (10). In view of Fig. 2(c), however, the 3-D baseline error will be only slightly higher than that given by the theoretical minimum of (12). In Fig. 2 the errors are shown as broken lines for those values of  $V_s$  and  $V_x$  resulting in  $\text{SNR} < 7$ .

The realizable minimum 3-D baseline errors due to system noise for the three different S-band system temperatures at the mobile site are shown in Fig. 1 as horizontal lines. It is seen that an increase in S-band system temperature at the mobile site by a factor of 4 increases the baseline error only slightly, provided that S/X data volume ratio is not far from optimum.

### III. Conclusions

The system temperature of the ORION S-band receiver has a very small effect on baseline determination accuracy. It is unwise to pay a high cost for an S-band receiver with a low system temperature; a system temperature as high as 240 K is tolerable. Only 20 to 40 percent of data volume is to be allocated to S-band observations for minimum baseline error. The error remains low over a rather wide range of data volume ratio. Hence precise allocation of the 14 pairs of the Mark III VLBI data channels between S- and X-band observations is not critical.

## Reference

1. Wu, S. C., "Error Estimation for ORION Baseline Vector Determination," *TDA Progress Report 42-57*, Jet Propulsion Laboratory, Pasadena, Calif., June 15, 1980, pp. 16-31.

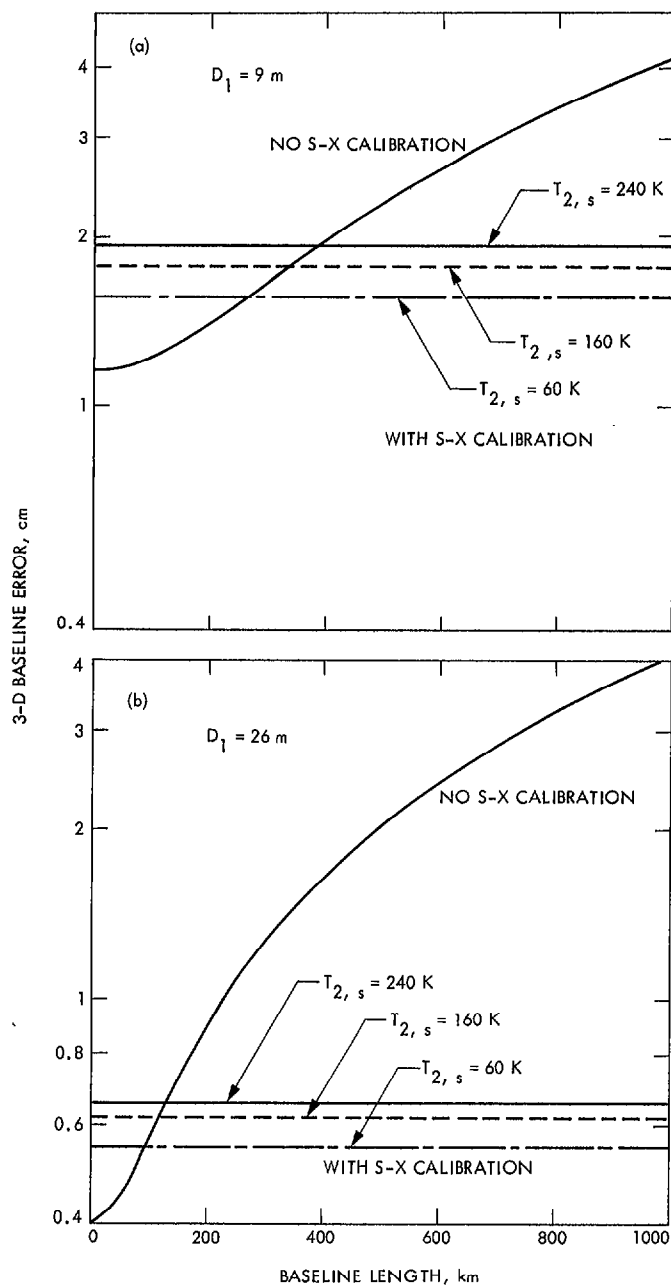


Fig. 1. RSS effects of system noise and ionosphere on baseline determination

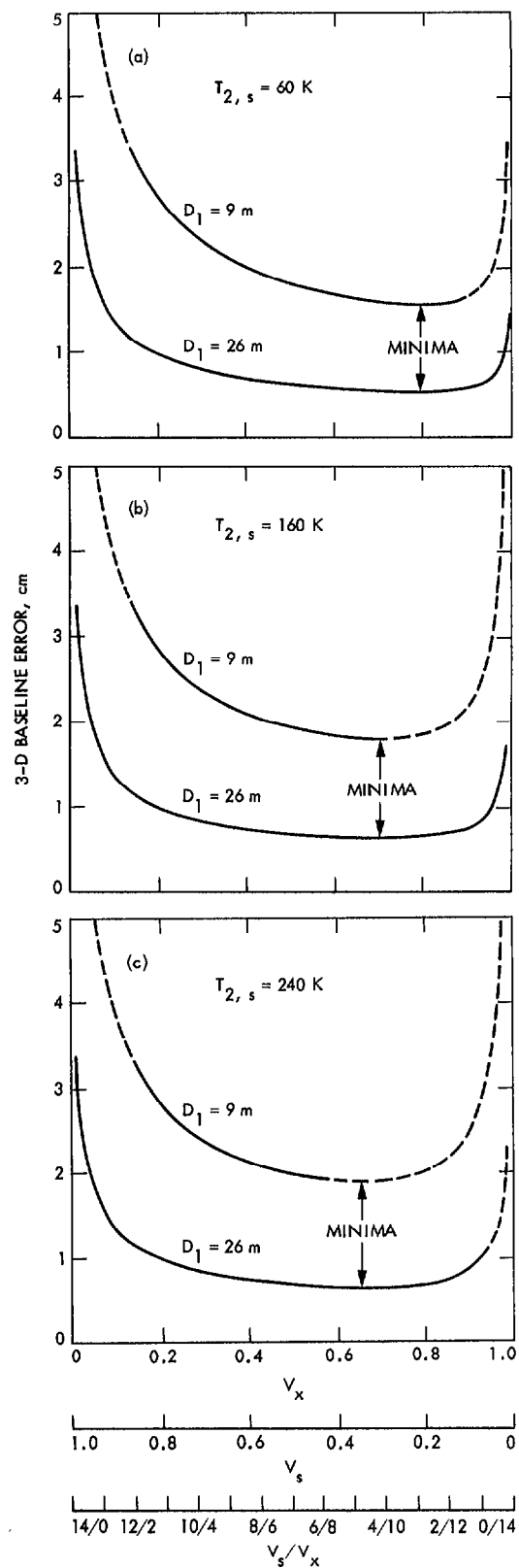


Fig. 2. Effects of S-X relative data volume on baseline errors due to system noise